Dependence of Adaptive Cross-Correlation Algorithm Performance on the Extended Scene Image Quality

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ABSTRACT

Recently, we reported an adaptive cross-correlation (ACC) algorithm to estimate with high accuracy the shift as large as several pixels between two extended-scene sub-images captured by a Shack-Hartmann wavefront sensor. It determines the positions of all extended-scene image cells relative to a reference cell in the same frame using an FFT-based iterative image-shifting algorithm. It works with both point-source spot images as well as extended scene images. We have demonstrated previously based on some measured images that the ACC algorithm can determine image shifts with as high an accuracy as 0.01 pixel for shifts as large as 3 pixels, and yield similar results for both point source spot images and extended scene images. The shift estimate accuracy of the ACC algorithm depends on illumination level, background, and scene content in addition to the amount of the shift between two image cells. In this paper we investigate how the performance of the ACC algorithm depends on the quality and the frequency content of extended scene images captured by a Shack-Hatmann camera. We also compare the performance of the ACC algorithm with those of several other approaches, and introduce a failsafe criterion for the ACC algorithm-based extended scene Shack-Hatmann sensors.

Keywords: Adaptive optics, Shack-Hartmann sensor, extended scene, remote imaging, wave-front sensing and control

1. INTRODUCTION

A Shack-Hartmann sensor (SHS) is an optical instrument widely used for wavefront sensing in optical testing and astronomical adaptive optics. It consists of a lenslet array and a camera. The lenslet array is placed in a plane conjugate to the plane of the wavefront error source, and the camera is located at the focal plane of the lenslet array. The conventional SHS uses a point-source such as a star, a laser or a pinhole as its imaging object. In such a case, the image captured by a Shack-Hartmann (SH) camera is an array of spot images, each of which is a sub-aperture PSF. When the wavefront error at the lenslet array changes, the position of each spot image at the SHS camera shifts from its original position, and one can measure the overall wavefront error by determining the shifts of all spot images from their reference positions. The location of each spot is usually determined by calculating the centroid (center of mass) of the spot [1].

In some applications, such as solar telescopes [2], remote-imaging along short horizontal or slant paths from the ground [3] and remote-imaging from space, a point-source is not available but the use of an SHS-based AO system is still desirable. In such a case, the SHS needs to image an extended scene and its lenslet array forms an array of sub-images in its camera. We call this type of SHS an extended-scene SHS (ES-SHS). In an ES-SHS, the wavefront error at the lenslet array shifts the sub-aperture images at the SH camera from their ideal locations. Therefore, the problem of measuring the wavefront error of a system reduces to the problem of estimating the shifts of all the sub-aperture images from their original positions. This is effectively a problem of image registration and the basic approach for solving this problem is cross-correlation [4]. This is the standard approach when the allowable transformations of the template include a small range of rigid transformations (translations, rotations and scale changes). In this approach, one typically computes the cross-correlation between a target image and a template for each allowable transformation of the template, and determines the types and the values of transformation parameters from the maximum of that cross-correlation function, as detailed in Ref. [4]. In this case, the cross-correlation between a target cell and a reference cell can be computed either in the spatial (or image) domain based on the Correlation theorem, or in the Fourier-domain using the Fast Fourier-Transform (FFT) [2, 3, 5, 6]. Recently, Knutsson et al. proposed a new approach which estimates the shift based on the phase of two images’ cross-correlation spectrum [7]. The approaches proposed by the researches cited above are restricted to situations where each lenslet image is shifted but almost undistorted and the mutual shift between

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two images is less than one pixel. Recently, we reported a new, adaptive cross-correlation (ACC) algorithm for ES-SHS \[8-9\]. It determines the shift between two image cells based on the phase of the two sub-images’ cross-correlation (CC) spectrum using an iterative image-shifting algorithm. The difference between this ACC algorithm and the one proposed by Knutson and Peterson \[7\] is that the latter uses only two phase terms and single iteration for the phase slope-finding, while the former uses 8 phase terms and multiple numbers of slope-finding iterations. Therefore, the former is slower but more accurate than the latter. The ACC algorithm works for both point-source spot images as well as extended scenes. By analyzing some image data measured from a single extended scene bar target on our ES-SHS testbed, we have shown that highly accurate shift estimates can be obtained with the ACC algorithm; the estimation error being less than 0.01 pixel when the actual shift between two image cells is greater than 3 pixels.

The performance of the ACC algorithm depends on the content of the scene as well as the illumination characteristics of the optical system such as illumination level and background light. Intuitively, it performs better when the scene has more features, more high-spatial-frequency content, and the Shack-Hatmann camera (SHC) receives more light. In this paper, we investigate the dependence of the ACC algorithm performance on the scene content, illumination level, and background light. The goal is to establish an acceptance criterion on image quality for guaranteeing a normal wavefront sensing operation with desired shift estimate accuracy. We will introduce a failsafe criterion, and also compare the performance of the ACC algorithm with those of two other faster approaches.

Section 2 of the paper is dedicated to the evaluation of the ACC algorithm performance when the illumination level and the background of an image change. Both simulated and measured images are used in the analysis of this section. Section 3 deals with the dependence of the ACC algorithm performance on extended scene content, and presents a highly reliable failsafe criterion. Section 4 compares the performance of the ACC algorithm with those of two other faster approaches. Finally, the paper is concluded with a brief summary in Section 5.

2. ILLUMINATION LEVEL AND BACKGROUND

The wavefront sensing accuracy of an ACC algorithm based ES-SHS is determined by the accuracy of the ACC algorithm, and the performance of the ACC algorithm depends on the illumination level, the background and the content of the extended scene, the noise characteristics of the Shack-Hartmann camera (SHC) as well as the shift between two image cells. Image quality and photon noise lead to noise on the wavefront slope estimates. The slope estimate noise can be measured with the bias and the variance of image shift estimates. In this section, we will examine the dependence of the ACC algorithm on the image illumination level and background. The dependency of the ACC algorithm on the image content will be described in the next section. The ACC algorithm has been detailed elsewhere \[8-9\] and will not be repeated here.

The extended scene we used to study the shift estimate noise of the ACC algorithm is a 500x500 pixel satellite photo shown in Fig. 1(a). The lenslet array in the SHS creates an array of sub-images at the SH camera, and each of these sub-images looks like an example shown in Fig. 1(b). This 50x50 pixel sub-image becomes much blurred as compared to the original scene, because its diffraction limit is defined by each lenslet array sub-aperture, not the whole aperture of the optical system. The ACC algorithm we implemented uses a 32x32 image cell for image-shifting and a smaller, 16x16 pixel cells for calculating the cross-correlation spectrum of the test and the reference cells. In Fig. 1(b), a 16x16 pixel and a 32x32 pixel areas are indicated by a black and a white frames, respectively. For our study in this paper, we normalized the image inside the circular sub-aperture in Fig. 1(b) such that its gray-scale is from 0 to 1.0. The gray-scale of the 32x32 pixel cell in Fig. 1(b) is from 0.063 to 0.966.

To analyze the extended scene shift estimation we use the following sub-image intensity model

\[ S_e(x, y) = I_0 \left[ I_b + \gamma_s G(x, y) \right] + N(x, y), \]

where \(G(x, y)\) is the normalized (0—1.0 gray-scale) 32x32 pixel cell as highlighted by a white-frame in Fig. 1(b), \(\gamma_s\) and \(\gamma_b\) are the signal and the background factors, respectively, \(I_0\) is the number of photons received by the brightest possible pixel in e/pix, \(N(x, y)\) is the noise term, and \(S_e(x, y)\) is a 32x32 sub-image cell measured in e/pix. The noise term includes \(G(x, y)\)–depended Poisson noise, detector read-out noise of 40 e/pix, and dark-current of 125 e/sec/pix. We modeled the read-out noise and the dark-current as added white Gaussian noise corresponding to an exposure time of 0.1 sec, and assumed \(I_0 = 50000\) e/pix in our simulations. Assuming a 12-bit SHC, we converted
Figure 1. (a) An example of an extended scene to be used in an extended-scene SH sensor. (b) Expanded view of one 50x50 pixel sub-image obtained from a simulated SH lenslet images. This sub-image is produced from the whole image shown in part (a). The white-frame shows a 32x32 pixel cell, and the black-frame corresponds to a 16x16 pixel cell.

Figure 2. (a) Dependence of shift estimate standard deviation ($\sigma$) on illumination level. The test and the reference cell images are obtained using Eqns. (1) and (2) with $\gamma_b = 0$ and with different noise distributions. In this case, varying $\gamma_s$ is the same as varying the illumination level. Each data point is obtained from 500 noise realizations. (b) Same as part (a) except $\gamma_b = 1 - \gamma_s$. In this case, varying $\gamma_s$ is equivalent to varying both the illumination level and the signal-to-background ratio. All of the noise terms are also included in this simulation. The 1/SNR values are shown in both figures as a reference. The original shift between the test and the reference cells are zero in both cases. That is, this simulation is for a zero-shift case. $\sigma$ is standard deviation.

$S_c(x,y)$ into $S_{dn}(x,y)$ measured in “digital-number” before data processing as follows:

$$S_{dn}(x,y) = round[S_c(x,y) \times 4095 / I_0],$$

where “round” represents a function for rounding a number towards the nearest integer and the subscript “dn” means “digital-number”.

Figure 2(a) shows the dependence of shift estimate standard-deviations, $\sigma_x$ and $\sigma_y$, on the illumination level. In this simulation, we set $\gamma_b = 0$ and varied $\gamma_s$ from 0.1 to 1.0. The test and the reference cells are obtained with different noise distributions, and each data point corresponds to 500 noise realizations. The curve of inverse signal-to-noise ratio, 1/SNR, is also included as a reference. The SNR is defined as [10]
Figure 3. (a) An SH camera image taken on the ES-SHS testbed at JPL. The window inside the white-frame is the active window of the SH camera to be used in this study. The Reference and the test cells indicated on this figure are the ones analyzed with the ACC algorithm in several cases in this paper. The gray-level of the SHC is 0 – 4095. (b) Eight frames were taken on the ES-SHS testbed using different exposure times as indicated in the title of each figure. Shown here are 64x64 pixel reference sub-images of those 8 frames with a red-frame indicating the central 16x16 pixel cell in each sub-image. They are all plotted in the same gray-scale for the purpose of comparison.

Figure 4. The values of the total shift estimate of 400 test cells relative to a reference cell. They correspond to the image frame with time = 39.7 msec and were obtained using the ACC algorithm. The shift values are plotted in an ascending order for clarity.

\[
\text{SNR} = \frac{\text{stdev}\{S(x, y) - N(x, y, G)\}}{\text{stdev}\{N(x, y, G)\}} = \frac{\sigma_s}{\sigma_n},
\]

where “stdev” represents a function for calculating standard-deviation and \(\sigma_s\) and \(\sigma_n\) are the standard deviations of the “noise-free” signal and the noise, respectively (The signal here can be considered as “noise-free” in the sense that we do not add any noise to the signal. However, the satellite photo shown in Fig. 1a already contains various detector noises). Figure 2(b) shows simulation results over a range of signal and background levels, or the effect of increasing the amount of background. The image cell used in this simulation, Fig. 1(b), has relatively good quality and performs fairly well as expected. The mean values of the shift estimates almost do not depend on the illumination level when \(\gamma_s \geq 0.2\) in the case of Fig. 2(a) or on background when \(\gamma_b \geq 0.4\) in Fig. 2(b), and their absolute values are less than 0.02 pixels in both cases. As is seen from both figures, the performance falls off with the inverse of the SNR, just as in the case of
Figure 5. (a) The average values of the 16x16 pixel cell maximum and minimum as well as the background outside the circular sub-aperture of the test cells as a function of exposure time. The 5x5 pixel area in the lower-left corner of each 64x64 pixel cell, as shown in Fig. 3(b), is used to calculate the mean “background” values. Each data point is the average of the parameter values of 400 cells. The gray-level of the SHC is 0 – 4095 and the data in this figure is divided by 4095. (b) Shift estimate standard deviations as a function of exposure time. They are calculated from $\delta x_{ki}' - \delta x_{ki} - \delta x_{bi}$ and $\delta y_{ki}' - \delta y_{ki} - \delta y_{bi}$, respectively. That is, the shift estimates corresponding to time = 35.7 msec ($i = 6$) are subtracted out from those corresponding to other 7 cases before calculating the standard deviations.

Figure 6. Same as Fig. 2(b) except $y_b = 2.5427 \gamma_s - 0.0392$. This relation is obtained based on the data shown in Fig. 5(a). The upper limit of $\gamma_s$ corresponds to $\gamma_s + \gamma_b \leq 1$ in both parts. (a) Actual shift between the test cell and the reference is zero (the zero-shift case). (b) Actual shift is 3 pixels in the x-direction. Each data point is the result of 500 noise realizations.

In Refs. [8-9], we tested the accuracy of the ACC algorithm by analyzing the sub-images of 8 frames taken on the ES-SHS testbed at Jet Propulsion Laboratory (JPL), California Institute of Technology. To confirm the results presented in Figs. 2, we chose 400 test cells and one reference cell from each of those 8 frames located inside the white-frame as shown in Fig. 3(a), and evaluate the relative shift estimate noise as a function of exposure time. Figure 3(b) shows the eight 64x64 pixel reference cells, one from each of the 8 frames. They are all plotted in the same gray-scale for the purpose of comparison. The exit pupil wavefront of our optical hardware was not fully corrected at the time of those 8...
“Periodic-Correlation” [5]. We have shown previously that shift estimate performance of the ACC algorithm almost does not frames were taken, so the different test cells in each frame showed different amount of shifts with respect to their reference cells. Figure 4 shows image shifts of the 400 test cells relative to their reference cell for the case of time = 35.7 msec. They were obtained with the ACC algorithm and were sorted in an ascending order for clarity. The “Total shift” is the radial shift value defined as $\delta_{r_{ki}} = \sqrt{\delta_{x_{ki}}^2 + \delta_{y_{ki}}^2}$, where $k = 1-8$ corresponds to the different exposure time values and $i = 1-400$ to the different test cells.

Figure 5(a) shows the average values of the maximum and the minimum of the 16x16 pixel cells as well as the sub-aperture background as a function of exposure time. The sub-aperture background labeled as “Background” is calculated by averaging the counts of the 5x5 pixel area in the lower-left corner of each 64x64 pixel cell as shown in Fig. 3(b), and each data point in this figure is the average of 400 data points corresponding to the 400 test cells. The “Minimum” in this figure roughly corresponds to $\gamma_s$, and the “Maximum – Minimum” to $\gamma_s$. As we can see, $\gamma_s \sim 0.1 – 0.2$ and $\gamma_b \sim 0.2 – 0.8$ over the range of exposure time. Figure 5(b) shows the standard deviations of $\delta x_{ki}' = \delta x_{ki} - \delta x_{0}$ and $\delta y_{ki}' = \delta y_{ki} - \delta y_{0}$, respectively. We chose the case of $k = 6$ (exposure time = 35.7 msec) as a reference because the signal is strongest in this case among the cases where no any pixel got saturated. The standard deviations of $\delta x_{ki}'$ and $\delta y_{ki}'$ are inversely proportional to the exposure time in the range of $k = 1 – 7$ except $k = 4$, and get worse after that because of the saturation of some pixels in the 16x16 pixel cells.

The result in Fig. 5(b) is obtained from the test cells whose initial shifts from their reference cells are as shown in Fig. 4. That is, different test cells are originally shifted from their reference cells by different amounts. To obtain better understanding about how an image having the characteristics as shown in Fig. 5(a) performs when the actual shift between the test cells and their reference is either zero or large, we first obtained the following image parameters from Fig. 5(a):

$$\gamma_b = 2.5427 \gamma_s - 0.0392, \quad \gamma_s + \gamma_b \leq 1.$$  \hspace{1cm} (4)

Then we repeated the analysis of Fig. 2(b) for the image in Fig. 1(b) using the parameters in Eq. (4) and including all of the noise terms. The results are shown in Fig. 6(a) for the zero-shift case and in Fig. 6(b) for the case where the original shift between the test cells and the reference is 3 pixels in the x-direction. The shift estimate noise is slightly worse in the case of actual x-shift = 3 pixels as compared to the zero-shift case as expected. However, given the large background-to-signal ratio (~2.5), it is fair to say that the shift estimate noise of the ACC algorithm is small. Also, the above results demonstrate once again that the performance of the ACC algorithm degrades very little when the actual shift between two image cells increases from zero to several pixels.

### 3. SCENE CONTENT AND A FAILSAFE CRITERION

The performance of the ACC algorithm depends strongly on the content of an extended scene. In the ideal case, a scene would produce an unbiased shift estimate with very low variance. In reality such an ideal scene is not always available and the ACC algorithm needs to work with differing scenes having different features and qualities. So it is important to understand how the performance of the ACC algorithm depends on extended scene content. Instead of conducting an exhaustive search on different scenes, we obtained more than 200 cells having a size of 32x32 pixels from the satellite image shown in Fig. 1(a), and produced from each a test and a reference cells as follows:

$$S_{im}(x,y) = R_i(x - ma, y),$$ \hspace{1cm} (5)

where $a$ is the width of a pixel, $m$ is an integer and $i$ is the cell index. That is, the test cell is the same as the reference cell except shifted by an integer number of pixels. Then we evaluated the shift estimates using the ACC algorithm for $m = 5$ and obtained the corresponding shift estimate errors, $\Delta x_i = |\tilde{x}_i - ma|$, for all of the cells. After that we evaluated the values of several image quality metrics of all the reference cells, and examined their relations to the $\Delta x_i$ values.

Considered image-quality metrics include the Mean-squared error and the modified Fisher-information [8], spatial-domain image sharpness [5], Fourier-domain visibility/contrast [7], and the RMS contrast [11]. However, we found that none of the above metrics as well as the approach we briefly described in Ref. [8] can serve as a failsafe criterion for the ACC algorithm when scene content changes to a large extent as in Fig. 1(a). After not being able to find a satisfactory
solution, we came up with a different failsafe criterion for the ACC algorithm on the scene content and quality. That is, we select a test and a reference cells from the same frame, preferably from two well-separated locations, such as shown in Fig. 3(a), shift the test cell by an integer number of pixels from \( m = -5 \) to 5 along the \( x \)-direction, and estimate the shifts using the ACC algorithm. We do the same in the \( y \)-direction as well. We accept an image frame if the maximum shift estimate errors (absolute values) in both the \( x \)- and the \( y \)-directions are smaller than a pre-determined limit, say, for example, 0.05 pixels. Otherwise we reject the image frame. We do this test in the earliest stage of the image data processing using the ACC algorithm. For an application that uses over 500 Shack-Hartmann sub-images, this image-quality test requires the processing of 22 additional pairs of test and reference cells. Therefore, in most applications this failsafe test does not add too much computational burden to a wavefront sensing (WFS) process, and, most importantly, it guarantees that an image frame passing this test will produce acceptable WFS results.

Figures 6(a) and 6(b) show the results of a failsafe test performed on the test and the reference cells similar to those shown in Fig. 3(a). The SHC image analyzed in this test is the one corresponding to an exposure time of 35.7 msec.

Three image shift errors shown in the figure legends are defined as follows:

\[
\begin{align*}
\Delta x_{mn} &= \delta x_{mn} - ma - \delta x_{m0} \\
\Delta y_{mn} &= \delta y_{mn} - na - \delta y_{m0} \\
\Delta r_{mn} &= \sqrt{\Delta x_{mn}^2 + \Delta y_{mn}^2}
\end{align*}
\]

where \( \delta x_{mn} \) and \( \delta y_{mn} \) are shift estimates obtained for the \( S_{mn}(x, y) = R(x - ma, y - na) \) pair, respectively. Figure 6(a) corresponds to \( n = 0 \) (\( x \)-shift), and Fig. 6(b) to \( m = 0 \) (\( y \)-shift). As we can see, all three shift estimate errors are less than 0.05 pixels everywhere in this example. The SH camera in our testbed had vertical gain non-uniformity which caused some artificial vertical dark-bright-dark-bright lines in the captured images. We believe such a vertical line feature in the captured image is responsible for the behaviors of the shift estimate error curves shown in Figs. 6.

### 4. COMPARISON OF DIFFERENT APPROACHES

In this section, we compare the performance of the ACC algorithm with those of two other faster approaches as well as of several faster versions of the ACC algorithm itself. The purpose is to identify a method for our application that gives high accuracy and as fast a speed as possible. The other two approaches we have examined are the Periodic-Correlation method detailed in Ref. [5] and the simpler version of the ACC algorithm proposed in Ref. [7], respectively. In the following, we call the approach of Ref. [7] as the “Two Phase Terms” method. The ACC algorithm produces a 16x16 pixel cross-correlation spectrum of a test and a reference cell with a nonlinear phase function, \( \phi(u, v) \). The digitized version of this phase function can be denoted as \( \phi(m, n) \) with \( m = 1, 2, \ldots, 16 \) and \( n = 1, 2, \ldots, 16 \), respectively. Its center is
Figure 7. (a) Radial shift estimate error, $\delta r_{\text{in}} - \delta r_{\text{out}}$, as a function of the actual shift in the $x$-direction, $\delta x$, where the subscripts “in” and “out” mean “input” and “output”, respectively. The reference cell is the same as in Fig. 1(b) and the test cell is obtained by shifting the reference cell in the $x$-direction using convolution. Only the digitization error is included in this simulation. (b) Shift estimate values of 362 cells relative to the corresponding reference cell. The SHC image is the one obtained with the exposure time of 35.7 msec (see Fig. 3b). The data points shown with the red-circles and the blue-squares are sorted in the same order as those shown with the green-curve. Only the results of the original 400 test cells whose shifts are less than 1 pixel are shown.

Figure 8. Effect of the slope-fitting iteration number of the ACC algorithm on the shift estimate accuracy, shown as $\delta r_{\text{in}} - \delta r_{\text{est}}$ versus $\delta x$. The sub-image used is the one shown in Fig. 1(b) and the test cell is obtained from the reference cell as $S_m(x, y) = R(x - ma, y)$. Only the digitization error is included in this simulation. (b) Same as Fig. 7(b) except that all the results are obtained with the ACC algorithm by only varying the number of phase slope-finding iterations.

at $m = n = 9$. The ACC algorithm uses 8 frequency components of $\phi(m, n)$ for a standard least-squares fit based slope-fitting, with $n = 9, 10, 11$ and $m = 9, 10, 11$, but excluding $m = n = 9$. As pointed out by Knutsson and Peterson [7] as well as some references herein, the above choice has the advantage of being the least sensitive to aliasing, and leaving the low spatial-frequency, high contrast components in the image but rejecting the high spatial-frequency, low SNR components completely from the shift estimate. The ACC algorithm also carries out the phase slope-fitting process multiple times until the incremental slope values are less than a tolerance or the maximum number of iterations is
reached. In the method of Ref. [7], only two terms, \( \phi_{i(9,10)} \) and \( \phi_{i(10,9)} \), are used for phase slope-fitting and the slope-fitting process is carried out only once.

To make sure that we implemented the Periodic-Correlation and the Two Phase Terms methods correctly, we first tested our computer codes with a pair of test and reference cells obtained as \( S_i(x, y) = R(x - \delta x_i, y) \), using the sub-image shown in Fig. 1(b) and varying the \( \delta x_i \) from 0 to 1.0 pixel with an increment of 0.04 pixel. The sub-pixel shift of the reference cell is obtained by convolution in the Fourier-domain, that is, from \( R(x - \delta x, y) = \mathcal{F}^{-1}\left\{ \mathcal{F}(R(u,v)) \exp[-j2\pi(\delta x)u] \right\} \), where \( \mathcal{F} \) denotes “Fourier-transform” and \( \mathcal{F}(R(u,v)) = \mathcal{F}\left\{ R(x, y) \right\} \). Figure 7(a) compares the radial shift estimate errors obtained with the three different methods. Here, the radial shift estimate error is defined as \( \delta r_{in} - \delta r_{out} \) with \( \delta r_s = \sqrt{(\delta x_s)^2 + (\delta y_s)^2} \) and \( s = in \) or \( out \) meaning “input” and “output”, respectively. In the simulations using the Periodic-Correlation and the Two Phase Terms methods, two 32x32 pixel sub-images were used for the test and the reference cells. The red-curve in this figure is almost the same as the result shown by the “Per. Corr.” curve in Fig. 1 of Ref. [5]. The shift estimate error obtained with the “Two Phase Terms” method (blue-curve) is almost zero. This is because the shift estimation process in this case uses the same sub-image size as in the image-shifting process from which the test cell is obtained and undoes what the image-shifting process did to the test cell. The ACC algorithm uses 16x16 pixel cells for phase slope-finding but yields less than 0.01 pixel shift estimate errors in the current case through multiple slope-finding iterations because the Tolerance of the ACC algorithm is set to 0.01 pixel.

We also compared the above three methods by analyzing the SHC image captured with an exposure time of 35.7 msec. The results are shown in Fig. 7(b). To obtain these results, 32x32 pixel cells were used in the method of Periodic-Correlation and 16x16 pixel ones in the case of Two Phase Terms to keep the latter consistent with the ACC algorithm. The result of the “ACC” case (green-curve) is the same as shown in Fig. 4 except that only the data points \( \leq 1 \) pixel are shown in this figure. The results of the other two methods are sorted in the same order as that of the ACC algorithm. As we can see from this figure, for the particular SHC image analyzed, the Periodic-Correlation method under-estimates the image shifts and the shift estimate values obtained with the Two Phase Terms approach also deviate randomly, and by large amounts in most cases, from those obtained with the ACC algorithm.

The ACC algorithm consumes less time if the number of phase slope-finding iterations is reduced. In order to understand the effects of the iteration number on the shift estimate accuracy, we conducted the following two simulations. In the first simulation, we used the sub-image in Fig. 1(b) for both the reference and the test cells obtaining the test cells from the reference as \( S_m(x, y) = R(x - ma, y) \), and estimated the shifts using the ACC algorithm with different number of iterations. The results are shown in Fig. 8(a). Only the digitization error is included in this simulation with \( I_0 = 50000 \) e/pix. The case “Adaptive” corresponds to the ACC algorithm and the mean number of iterations used in this case is 5.4. These results clearly demonstrate the important role played by the “adaptive” feature of the ACC algorithm. In the second simulation, we conducted a similar analysis on a set of image cells measured on our ES-SHS testbed with an exposure time of 35.7 msec (see Fig. 3b). Figure 8(b) compares the shift estimate results obtained with the above four different approaches. The results of all the other approaches were sorted in the same order as the “Adaptive” case. The red-curve is the same as shown in Fig. 4 and the mean number of iterations required by this standard ACC algorithm is 4.2. As we can see, using an iteration number of 2 yields much better results than the case of “Iter. Number = 1” in the present case of small image shifts—the largest image shift in the “Adaptive” case is less than 1.5 pixels. This result shows the importance of keeping the value of the total number of phase slope-finding iterations flexible in the ACC algorithm. By adjusting it according to the quality of the SHC image and the expected maximum value of image shifts in an application, one can decrease the image processing time while still obtaining satisfactorily high shift estimate accuracy. This result also shows that the standard ACC algorithm needs to be used to guarantee high accuracy when the maximum value of image shifts is much larger than one pixel.

5. CONCLUSION

This paper has presented a study about the dependence of the Adaptive Cross-Correlation (ACC) algorithm performance on the quality and the content of an extended scene image. Image quality parameters considered for numerical analysis included illumination level, background, Poisson noise, read-out noise, dark-current and digitization error. Numerical predictions about the effects of illumination level and background were obtained by combining a simulated Shack-Hatmann camera sub-image with a Poisson statistical model, and predicted results were confirmed with the results of the
analysis of measured SHC extended scene images. Our analysis has shown that, for the case of zero-shift, the shift estimation performance of the ACC algorithm follows an inverse power law in SNR for error standard deviation, confirming the previous findings of other researchers on both point source and extended scene. It was also shown that the shift estimation error standard deviation stays well below 0.1 pixel for a large range of illumination level and background when a sub-image is noisy but has reasonable level of frequency content. The measured SHC images had much less spatial feature than the simulated one, but still exhibited good performance in terms of shift estimation noise.

An image quality acceptance test was proposed. It serves as a failsafe criterion for the ACC algorithm by taking care of all of the quality aspects of either a point source or an extended scene image, including illumination level, background, detector noise, image content, sampling format, field of view, etc. Once a Shack-Hartmann camera image passes this test, the ACC algorithm will provide image shift estimates with acceptable accuracy for the estimation of wavefront. Therefore, this acceptance test can be considered as a new, highly valuable feature of the ACC algorithm.

It was shown previously that when using only two frequency components in the cross-correlation spectrum and single slope-finding iteration (“Two Phase Terms” method), the precision of the shift estimate was comparable to the conventional method of localization of the spatial-domain cross-correlation peak by parabolic interpolation (“Periodic-Correlation” method). The performances of these two methods were compared with that of the ACC algorithm. When analyzing the same set of sub-images obtained on our ES-SHS testbed whose shifts from their reference cell ranged from zero to 1.0 pixel, the Periodic-Correlation method under-estimated the shift values as compared to the ACC algorithm. The shift values obtained using the Two Phase Terms method also deviated by large, random amounts from those obtained with the ACC algorithm. The effects of using less number of phase slope-finding iterations in the ACC algorithm were also examined. It was found that one can achieve acceptable shift estimate accuracy using only 2 to 3 slope-finding iterations when the values of image shifts do not exceed too much from one pixel. If the image shifts are much larger than one pixel, one should use the standard ACC algorithm to achieve high shift estimate precision.

The material presented in this paper enables the better understanding of the ACC algorithm and its more robust implementation in AO systems that conduct Shack-Hartmann-based wavefront sensing using either a point source or arbitrary extended scenes.

REFERENCES